84

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Section: 2

Exam II

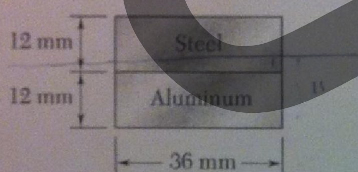
Thursday May 12, 2011

All work **must** be shown to receive full credit.

Problem 1. (20 pts.) A steel bar (E_s = 210 GPa) and an aluminum bar (E_a = 70 GPa) are bonded together to form the composite bar shown. When the bar is bent about a horizontal axis, with M = 200 $N.m$, determine:

a - The maximum stress in the aluminum.

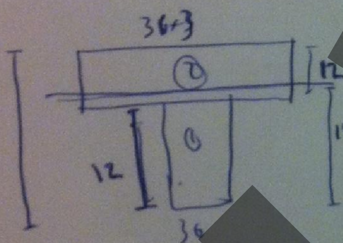
b - The maximum stress in the steel.



$n = \frac{E_s}{E_a} = \frac{210}{70} = 3$ 3

4

$\bar{y} = \frac{36\times12\times6 + 3\times36\times12\times18}{36\times12 + 36\times3\times12} \Rightarrow \bar{y} = 15\,mm = 0.015\,m$



$I = I_1 + I_2 = \frac{1}{12}(0.036)(0.012)^3 + 0.036\times0.012(0.006-0.015)^2$
$+ \frac{1}{12}(0.036)\times3\times(0.012)^3 + \cancel{3\times36\times}\ 3\times0.036\times0.012\times(0.018-0$

$\Rightarrow I = \cancel{8.1\times10^{-9}} + \cancel{1.6524\times10^{-8}} \Rightarrow \cancel{I = 2.46\times10^{-8}}\ m^4$
$4.017\times10^{-9}\ \ 2.7\times10^{-9} \Rightarrow I = 6.7\times10^{-9}\,m^4$ 1

a) $|\sigma_A| = \left|\frac{MC}{I}\right| \Rightarrow \sigma_A = \frac{200\times0.009}{\cancel{2.46\times10^{-8}}\times6.7}$ 3 $(C_{ep} = 0.009m)$

$\cancel{\sigma_A = 73.17\ MPa}$ $\sigma_A = 26.86\,MPa$

b) $|\sigma_s| = \left|\frac{nMC}{I}\right| = \frac{3\times200\times0.003}{\cancel{2.46}\times10^{-9}\times6.7}$ 3 $(C_s = 15-12 = 3mm)$

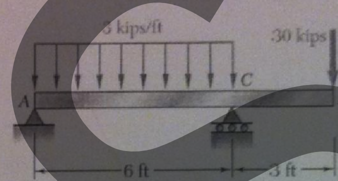
$\cancel{\sigma_s = 73.17\ MPa}$ $\sigma_s = 26.86\,MPa$

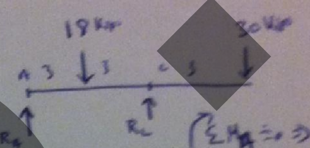
11

Problem 2. (20 pts.) Draw the shear and bending moment diagrams for the beam and loading shown, and determine:

a - The maximum absolute value of the shear

b - The maximum absolute value of the bending moment.





$\curvearrowleft \sum M_C = 0 \Rightarrow$

$30 \times 3 - 18 \times 3 + 6 R_A = 0$

$\Rightarrow R_A = -6$ kips $= 6$ kips $\downarrow$ 2

$\curvearrowleft \sum M_A = 0 \Rightarrow 18 \times 3 - 6R_C + 30 \times 9 = 0$

$\Rightarrow R_C = 54$ kips 2

$V_A = R_A = -6$ kips

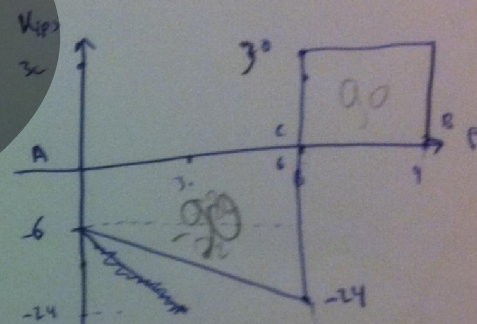
$V_C^- = V_A - \int w\,dx = -6 - 18 = -24$ kips

$V_C^+ = V_C^- - \int w\,dx + R \Rightarrow V_C^+ = -24 + 54 = 30$ kips

~~$V_B = 30$~~ $V_B = V_C^+ - R_B = 30 - 30 = 0$

a) max shear is 30 kips.

2



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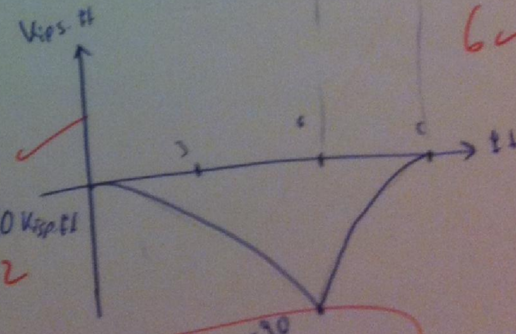

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$M_A = 0$

$M_C = M_A + \int V\,dx = -\frac{(24+6)\,6}{2} = -90$ kips.ft

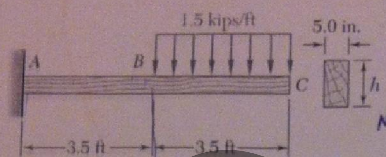
$M_B = M_C + \int V\,dx = -90 + 30 \times 3 = -90 + 90 = 0$

b) max absolute bending moment is 90 kips.ft

2

Problem 3. (20 pts.) For the beam and loading shown, design the cross-section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 *psi*.

$\sigma_{all} = 1750$



5.25

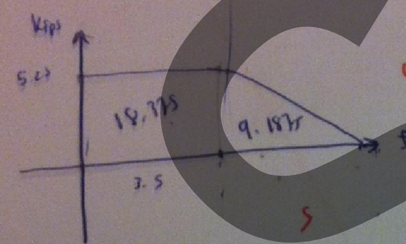
A 3.5 B 1.75 ↓ 1.75 C

M_A R_A $\Sigma M_A = 0 \Rightarrow 5.25 \times (1.75 + 3.5) + M_A = 0$

$M_A = -27.56$ kips.ft $= 27.56$ kips.ft

$R_A = 5.25$ kips

2 2



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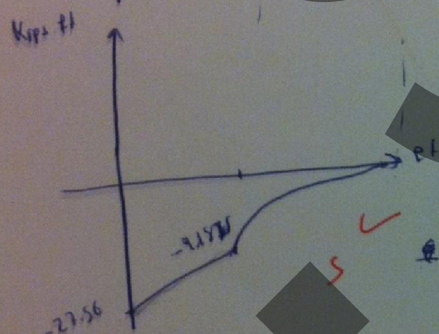


5

$|M_{max}| = 27.56$ Kips.ft $= 330.72$ Kips.in

$\sigma = \left| \frac{Mc}{I} \right|$

$I = \frac{1}{12} h^3 \times 5$

$c = \frac{h}{2}$

$\frac{c}{I} = \frac{\frac{h}{2}}{\frac{1}{12} h^3 \cdot 5} = \frac{6}{h^2 5}$

$\sigma = M \cdot \frac{6}{h^2 5} \rightarrow h^2 = \frac{6M}{5\sigma}$

$\Rightarrow h^2 = \frac{6 \times 330.72 \times 10^3}{5 \times 1750}$ 4

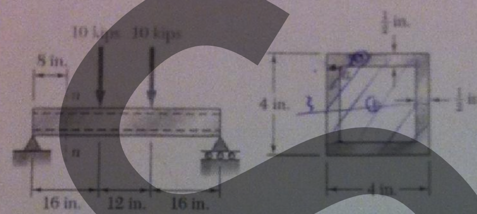
$\Rightarrow \boxed{h = 15.06 \text{ in}}$

20

Problem 4. (20 pts.) For the beam and loading shown, consider section n – n and determine:

a - The largest shearing stress in that section.

b - The shearing stress at point *a*.



$\bar{y} = 2\,in$

$I = I_2 - I_1$

$= \frac{1}{12} \times 4 \times 4^3 - \frac{1}{12} \times 3 \times 3^3 \Rightarrow I = 14.583\ in^4$.

a) $\tau = \frac{VQ}{It}$

$V = 10$ kips

$I = 14.583\ in^4$

$t = 4\,in$

$Q = 2 \times (4 \times 4 - 3 \times 3) = 14\,in^3$

2

2 4

$\tau_{max} = \frac{10 \times 10^3 \times 14}{4 \times 14.583} \Rightarrow \tau_{max} = 2400\ psi$

b) $\tau_a = \frac{VQ}{It}$

$V = 10$ kips

$I = 14.583\ in^4$

$t = \cancel{4\ 0.5}\ in$

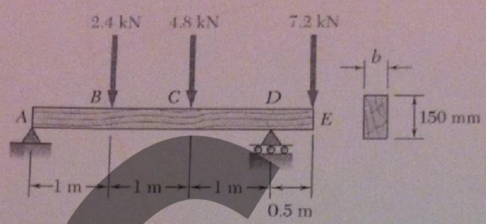
$Q = \frac{1}{2} \times 4 \left(2 + \frac{1}{4}\right) in^3$

$Q = 4.5\ in^3$

4

$\tau_a = \frac{4.5 \times 10 \times 10^3}{14.583 \times 4\,in} \Rightarrow \tau_a = 771.44\ psi$.

Problem 5. (20 pts.) For the beam and loading shown, determine the minimum required width b, knowing that $\sigma_{all} = 12\ MPa$ and $\tau_{all} = 825\ kPa$.

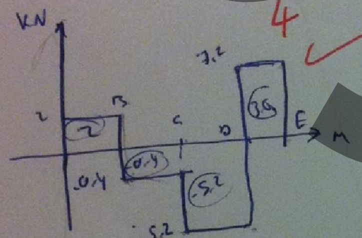


~~$\curvearrowright \Sigma M_A = 0 \Rightarrow 2.4 + 4.8 \cdot 2 + 7.2 \ldots R_D \Rightarrow$~~
~~$R_D = 14.4\ kN$~~
~~$\Sigma F \uparrow = 0 \Rightarrow$~~

$\curvearrowright \Sigma M_A = 0 \Rightarrow 2.4 + 4.8 \times 2 + 7.2 \times 3.5 - 3R_D = 0 \Rightarrow R_D = 12.4\ kN$ ✓

$\curvearrowright \Sigma M_D = 0 \Rightarrow 3R_A - 2.4 \times 2 - 4.8 + 7.5 \times 0.5 = 0 \Rightarrow R_A = 2\ kN$ 2

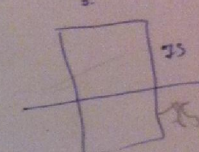
Check: $\Sigma F \uparrow = 0 \Rightarrow 12.4 + 2 - 2.4 - 4.8 - 7.2 = 0$ ✓ $0 = 0$



4 ✓

$* \ \tau = \frac{QV}{It}$

$Q = 0.075b \cdot \frac{0.075}{2}$
$Q = 2.81 \times 10^{-3}\ b\ m^3$ 2
$V = 7.2\ kN$
~~$I = \frac{1}{12} b (0.075)^3$~~ ~~$I = \frac{1}{12} b (0.15)^3$~~ $I = \frac{1}{12} b (0.075)^3$
$t = b$
$\tau = 825\ kPa$



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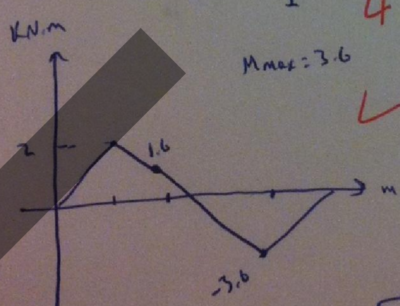
$\tau = \frac{2.81 \times 10^{-3}\ b \times 7.2 \times 10^3 \times 12}{b^2 \times (0.075)^3} \Rightarrow b = \frac{2.18 \times 12 \times 7.2}{825 \times 10^3 \times (0.075)^3}$ ~~$\Rightarrow b = 5.4 \times 10^{-4}\ m$~~

~~$b = 0.54\ mm$~~ ~~$b = 0.7\ mm$~~

$\boxed{b = 0.54\ m}$

$* \ \sigma = \frac{Mc}{I}$

4 $\frac{c}{I} = \frac{0.075}{\frac{1}{12} b \times 0.15^3} = \frac{0.075 \times 12}{0.15^3\ b}$



$M_{max} = 3.6$ ✓

4

$\sigma = M \cdot \frac{12 \times 0.075}{0.15^3\ b}$

$\Rightarrow b = \frac{12 \times 3.6 \times 10^3 \times 0.075}{12 \times 10^6 \times 0.15^3}$ $\Rightarrow b =$ ~~$1.7 \times 10^{-4}\ m$~~ ~~$0.08\ m$~~ ~~$b = 0.17\ mm$~~ ~~$b = 0.08\ m$~~

$\Rightarrow b = 0.64\ m$ ✓

we take the largest b

$\Rightarrow$ ~~$b = 0.54\ mm$~~

$\boxed{b = 0.08\ m}$

then the minimum b required is 0.54 m

18

92

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Section: 31

Exam II

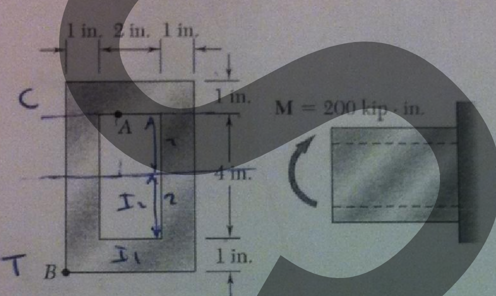
Tuesday May 11, 2010

All work **must** be shown to receive full credit.

Problem 1. (20 pts.) Knowing that the couple shown acts in a vertical plane, determine the stress at:

a - Point *A*.

b - Point *B*.



$$I = I_1 - I_2$$

$$I_1 = \frac{bh^3}{12} = \frac{4 \times 6^3}{12} = 72 \text{ in}^4$$

$$I_2 = \frac{2 \times 4^3}{12} = 10.6667 \text{ in}^4$$

$$\boxed{I = 61.333 \text{ in}^4}$$

$$\sigma = -\frac{Mc}{I}$$

$$\frac{\text{kipin} \times \text{in}}{\text{in}^4} = \frac{\text{kip}}{\text{in}^2} = \text{psi}$$

• $\sigma = \frac{Mc}{I}$

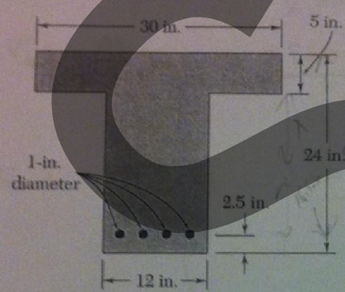
a) $\sigma_A = -\frac{200 \times 2}{61.333} = -6.5217$ psi

b) $\sigma_B = \frac{200 \times 3}{61.333} = 9.7826$ psi

Problem 2. (20 pts.) Knowing that the bending moment in the reinforced concrete beam is 150 *kip.ft* and that the modulus of elasticity is 3.75×10^6 *psi* for the concrete and 30×10^6 *psi* for the steel, determine: *(Hint: remember that 1 ft = 12 in)*

a - The stress in the steel.

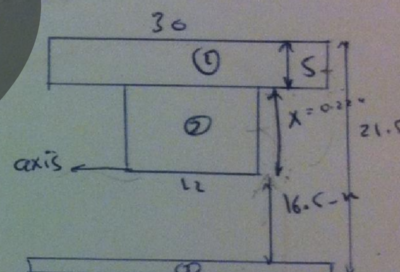
b - The maximum stress in the concrete.



Concret	steel
$E = 3.75\times10^6$ psi	$E = 30\times10^6$ psi

$M = 150$ Kip.ft

$\boxed{M = 1800 \text{ Kip.in}}$



$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3.75\times10^6} = 8$$

- $A_{steel} = 4\times\frac{\pi\times1^2}{4} = 3.1415 \text{ in}^2$
- $nA = 25.13274 \text{ in}^2$

- $A\bar{y}$
- $(30)\times(5)(x+2.5) + (12)(x)(\frac{x}{2}) - 25.13274(16.5-x)$

$150x + 375 + 6x^2 - 414.69021 + 25.13274x$

$175.13274x + 6x^2 - 39.69021$

$\boxed{x = 0.22489} \text{ in}$

a) $\sigma = \frac{Mc}{I}$

$I_1 = \frac{bh^3}{12} + Ad^2$

$= \frac{30\times5^3}{12} + (30\times5)(2.72489)^2 = 1426.2538 \text{ in}^4$

$I_2 = \frac{12\times0.22489^3}{12} + (12)(0.22489)\left(\frac{0.22489}{2}\right)^2 = 0.07961 \text{ in}^4$

very small

$I_3 = \cancel{\frac{bh^3}{12}} + 25.13274\times(16.27511)^2 = 6657.1402 \text{ in}^4$

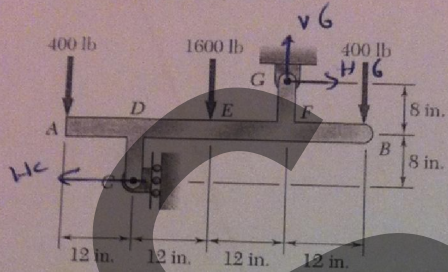
$= \cancel{409.038 \text{ in}^4}$

18

Problem 3. (20 pts.) Draw the shear and bending moment diagram for the beam and loading shown, and determine the maximum absolute value:

a - of the shear.

b - of the bending moment.



• $\Sigma M_G = -400(36) - 1600(12) + HC(16) + 400(12)$

$\Rightarrow \boxed{HC = 1800\,lb}$

• $\Sigma F_x = HG - HC = 0$

$\boxed{HG = 1800\,lb}$

• $\Sigma F_y = -400 - 1600 - 400 + VG = 0$

$\boxed{VG = 2400\,lb}$

• $M_D = HC \times 8 = 1800 \times 8$
$= 14400$

• $M_F = HG \times 8 = 14400$

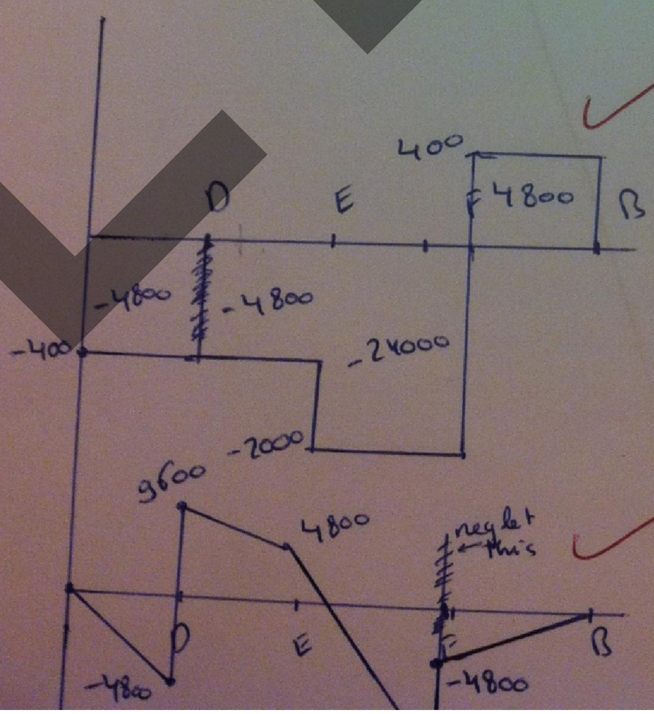
• $M_D = M_A + (-400 \times 12)$

• $M_D^- = -4800$
$M_D^+ = -4800 + 14400 = 9600$

$M_E = M_D + (-4800)$
$= 9600 - 4800 = 4800$

• $M_F^- = M_E - 24000 = -19200$
$M_F^+ = -19200 + 14400 = -4800$

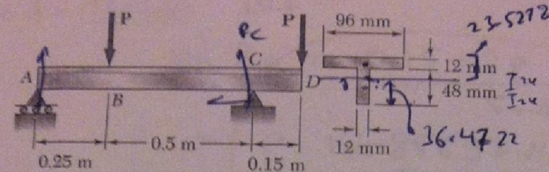
• $M_B = -4800 + 4800 = 0$



a) $\tau_{max} = 2000$ lb in

b) $M_{max} = 19200$ lb in

Problem 4. (20 pts.) Determine the largest permissible value P for the beam and loading shown, knowing that the allowable normal stress is 80 MPa in tension and −140 MPa in compression.



$\sigma_{all} = 80$ Mpa (T)

$\sigma_{all} = -140$ Mpa (C)

$\bullet\ \Sigma M_A = 0.25P + 0.9P - Rc(0.75)$

$1.15P - Rc(0.75) = 0$

$\boxed{Rc = 1.5333\,P}$

$\bullet\ \Sigma F_y \uparrow = Rc + RA - P - P = 0$

$\boxed{RA = 0.4666\,P}$

$\bar{Y}$: $\bullet\ Y_1 = 24$ mm $\quad Y_2 = 54$ m

$A_1 = 576$ mm² $\quad A_2 = 1152$ mm

$\bar{Y} = \dfrac{(24)(576) + (54)(1152)}{576 + 1152}$

$\boxed{\cancel{\bar{Y} = 36.4722 \text{ mm}}}$ $\quad \bar{Y} = 44$

$\bullet\ I = I_1 + I_2$ back 2nd page

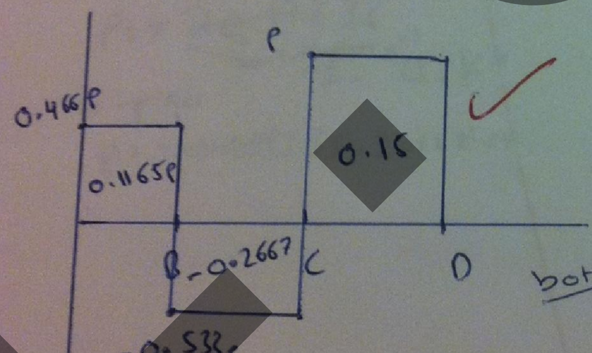
$I_1, I_2 = \frac{bh^3}{12} + Ad^2$

$I_1 = \frac{12 \times 48^3}{12} + 576(12.4722)^2$

$\boxed{I_1 = 20.0192 \times 10^4 \text{ mm}^4}$

$I_2 = \frac{96 \times 12^3}{12} + 1152(17.527$

$\boxed{I_2 = 36.77457 \times 10^4 \text{ mm}^4}$



$\circ\ M_B = \cancel{M_A}^{0} + 0.1165P$

$\circ\ M_C = M_B + (-0.2667) = -0.1502P$

$M_D = M_C + 0.15 = 0$

$I = I_1 + I_2 = 56.79377 \times 10^4$ mm

at B

bottom case 1: tension: $\sigma = \dfrac{Mc}{I}$

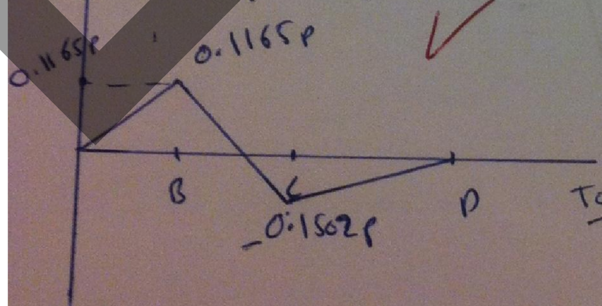
$10^3 \times 80 = \dfrac{0.1165P \times \cancel{36.4722} \times 10}{\cancel{56.79377 \times 10^{-7}}}$

KN/m²

4.7001×10

$P_1 = \cancel{10.69367 \text{ KN m}^2}$

$\boxed{P_1 = \cancel{7.399}}$ (6a $\bar{Y}$)



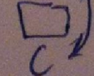
Top: case 2: Compressi

$10^3 \times 140 = \dfrac{0.1165P \times (23.5278)}{\cancel{56.79377 \times 10^{-7}}}$

4.70016×10

$P_2 = \cancel{29.008 \text{ KN}}$

$\boxed{P_2 = \cancel{24.0067} \text{ KN}}$

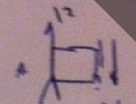


$P_2 = 35.2016$ ⇒ cont

Problem 5. (20 pts.) For the beam and loading shown, consider section n − n and determine the shearing stress at:

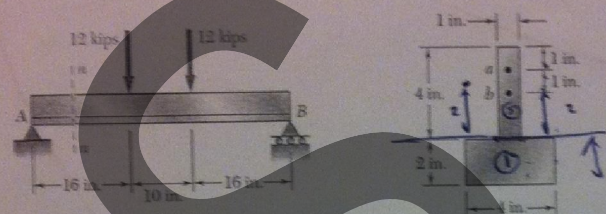
a - Point a.

b - Point b.



$\Sigma F_y = 12 - V = 0$ $\boxed{V = 12}$ Kips

$V = 12$



$\Sigma M_A = (12)(16) + (12)(26) - R_B(42) = 0$

$\boxed{R_B = 12 \text{ Kips}}$

$\Sigma F_y = R_A - 12 - 12 + R_B = 0$

$\boxed{R_A = 12 \text{ Kips}} = V$

$Y_1 = 1$ in $\quad Y_2 = 4$ in

$A_1 = 8$ in² $\quad A_2 = 4$ in²

$$\bar{Y} = \frac{(1)(8) + (4)(4)}{8+4} = 2 \text{ in}$$

$I = I_1 + I_2$

$I_1 = \frac{(4)(2)^3}{12} + (4)(2)(1)^2 = 10.667 \text{ in}^4$

$I_2 = \frac{1 \times 4^3}{12} + (4 \times 1)(3)^2 = 41.333 \text{ in}^4$ $\quad Q = A\bar{y}$

$I = 51.9999 \text{ in}^4$

- $Q_a = 1 \times 3.5 = 3.5 \text{ in}^3$
- $Q_b = 2 \times 2.5 = 5 \text{ in}^3$

a) $\tau = \frac{VQ}{It} = \frac{12 \times 3.5}{51.999 \times 1}$ (Kips, in³, in⁴ × in)

$\tau_a = \cancel{0.8...}$ psi 0.807693 psi

b) $\tau_b = \frac{12 \times 5}{51.99 \times 1} = \cancel{0.923078 \text{ psi}}$

$= 1.153846$ psi.

14

Name: Abbas Zein
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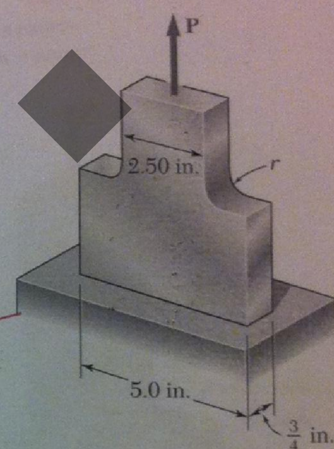
Section: 31

~~8~~ 4

Quiz 2

Knowing that P = 10 kips, determine the maximum stress when r = 0.5 in.



$k = \frac{\sigma_{max}}{\sigma_{aver}}$

$\sigma_{max} = K \times \sigma_{aver}$

$\sigma_{ave} = \frac{P \text{ kips}}{A \text{ in}^2}$

1- $\boxed{A = \frac{3}{4} \times 2.5 = 1.875 \text{ in}^2}$

$\boxed{\sigma_{ave} = \frac{10}{1.875} = 5.3333}$ $\boxed{\frac{D}{d} : 2}$

2 $\frac{r}{d} = \frac{0.5}{2.5} = 0.2$

$\boxed{K = \cancel{1.9}}$ 1.95

$\sigma_{max} = 1.95 \times 5.333$

$\boxed{\sigma_{max} = \cancel{3.4662}}$

$\sigma_{max} = 0.399535$ Kips/in²